

Papiro 04
MATEMÁTICA

1ª QUESTÃO	Valor 1,0
Find all natural numbers n such that n^2 does not divide $n!$.	
2ª QUESTÃO	Valor 1,0
Can both $x^2 + y$ and $x + y^2$ be squares for x and y natural numbers?	
3ª QUESTÃO	Valor 1,0
Find all polynomials $p(x)$ of degree $\leq n$ which satisfy $p(x^2) = p(x)^2$ for all real x .	
4ª QUESTÃO	Valor 1,0
P is a point inside an equilateral triangle. Its distances from the vertices are 3, 4, 5. Find the area of the triangle.	
5ª QUESTÃO	Valor 1,0
What is the largest power of 10 dividing $100 \times 99 \times 98 \times \dots \times 1$?	
6ª QUESTÃO	Valor 1,0
What is the remainder on dividing $x + x^3 + x^9 + x^{27} + x^{81} + x^{243}$ by $x - 1$? By $x^2 - 1$?	

7ª QUESTÃO**Valor 1,0**

a, b are the roots of $x^2 + Ax + 1 = 0$, and c, d are the roots of $x^2 + Bx + 1 = 0$. Prove that $(a - c)(b - c)(a + d)(b + d) = B^2 - A^2$.

8ª QUESTÃO**Valor 1,0**

Prove:

$$\left(\binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \dots \right)^2 + \left(\binom{n}{1} - \binom{n}{3} + \binom{n}{5} - \dots \right)^2 = 2^n.$$

9ª QUESTÃO**Valor 1,0**

Consider the cube root of unity

$$\varepsilon = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}.$$

Compute

$$(1 + \varepsilon)(1 + \varepsilon^2) \dots (1 + \varepsilon^{1987}).$$

10ª QUESTÃO**Valor 1,0**

$f(x)$ is a polynomial of degree n such that $f(0) = 0$, $f(1) = 1/2$, $f(2) = 2/3$, $f(3) = 3/4$, ... , $f(n) = n/(n+1)$. Find $f(n+1)$.